Gauged R-symmetry, Fermion and Higgs Mass Problem

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Abstract

We consider the simplest model of $SU(3) \times SU(2) \times U(1)_Y \times U(1)_R$ gauge symmetry with one extra singlet field whose vacuum expectation value breaks the horizontal R-symmetry $U(1)_R$ and gives rise to Yukawa textures. The $U(1)_R$ symmetry is able to provide both acceptable fermion mass hierarchies and a natural solution to the μ problem only if its mixed anomalies are cancelled by the Green-Schwarz mechanism. When the canonical normalization $g_3^2 = g_2^2 = \frac{5}{3}g_1^2$ of the gauge coupling constants is assumed, the Higgs mass parameter $\mu \sim m_{3/2}$ can arise taking into acount the uncertainty in the ultraviolet relation $m_e m_\mu m_\tau / m_d m_s m_b \simeq \lambda^q$ with $q \neq 0$. When $q = 0$ is taken only a suppressed value of $\mu \sim \lambda m_{3/2}$ can be obtained.

arXiv:hep-ph/9508320v2 18 Aug 1995

Recently gauged horizontal $U(1)$ symmetries have been considered as an appealing tool for understanding fermion mass problem [1]–[5]. In the simpliest model with one extra $SU(3) \times SU(2) \times U(1)_Y$ singlet field, acceptable mass matrices arise if anomalies of a horizontal $U(1)$ gauge symmetry is cancelled by the Green-Schwarz mechanism [6]. Remarkably the Green-Schwarz cancellation can predict the ultraviolet value of the weak mixing angle $\sin^2 \theta_W = 3/8$ [7] without being related to grand unified groups.

Another persistent question in the minimal supersymmetric standard model is the μ-problem concerning the Higgs mass whose origin should be related to physics beyond the standard model. Even if supersymmetry can stabilize Higgs masses it alone does not explain why the μ-term $\mu H_1 H_2$ allowed by the standard model gauge symmetry has such a small parameter μ compared to e.g. the Planck mass: $\mu \ll M_P$. It was pointed out [8, 9] that the horizontal $U(1)$ symmetry yielding fermion mass hierarchies can resolve the μ-problem by forbidding the appearance of the direct μ-term in superpotential and allowing its effective generation through supersymmetry breaking [1].

In this brief letter, we discuss how a horizontal R-symmetry $U(1)_R$ can provide both fermion mass hierarchies and a natural generation of the μ-term. It turns out that being compatible with fermion mass matrices $U(1)_R$ has to be anomalous like non-R horizontal symmetry. Therefore, its spontaneous breaking gives rise to an unwanted axion unless horizontal R-symmetries are gauged through the Green-Schwarz mechanism. Gauged R-symmetry requires supersymmetry to be local since R-symmetry does not commute with supersymmetry.

In the framework of local supersymmetry (supergravity) endowed with R-invariance, the μ-problem can be resolved in a natural way [12]. If the Higgs mass term $H_1 H_2$ carries R-charge zero, it can appear only in Kähler potential:

$$K \sim H_1 H_2 \ , \tag{1}$$

together with the usual kinetic terms of $H_{1,2}$. This leads to the effective μ-term in superpotential

$$W_{eff} \sim \frac{W}{M_P^2} H_1 H_2 \ . \tag{2}$$

In this way the appearance of the μ-parameter is an inevitable consequence of breakdown of both supersymmetry and R-symmetry, through which the value of μ is determined by the

[1]The anomalous global Peccei-Quinn symmetry introduced to solve the strong-CP problem can also be used to explain the appearance of the μ-term [10, 11].

gravitino mass $m_{3/2} \equiv \langle W \rangle / M_P^2 : \mu \sim m_{3/2}$. Appearance of such a term in effective super-potential was discussed in general supergravity theory [13] and in the context of superstring theory [14]. This situation has to be contrasted to the cases with non-R horizontal symmetry where the μ-term has to appear through non-renormalizable terms in Kähler potential. In this case the μ-parameter always has a suppression by factors of the Cabibbo angle $\lambda \sim 0.2$: $\mu \sim \lambda^k m_{3/2}$ with $k = 1, 2, \cdots$ [8, 9].

Let us now discuss the conditions on horizontal $U(1)_R$ charges yielding acceptable fermion mass matrices. We assume the simplest case with only *one* expansion parameter λ resulting from the vacuum expectation value of an extra singlet χ: $\lambda \sim \langle \chi \rangle / M_P$. Let us denote the R-charges of i-th family quarks and leptons in terms of the obvious notations: q_i, u_i, d_i, l_i and e_i. The R-charges of the corresponding squarks and sleptons are then given by $q_i + 1$, etc.. We assign the R-charges $h_{1,2}$ for the Higgs fields $H_{1,2}$. Therefore, the Higgsinos have the R-charges $h_{1,2} - 1$. The choice (1) fixes $h_1 + h_2 = 0$. The R-charge of the singlet is denoted by $-r_\chi$. The superpotential of quarks and leptons invariant under $SU(3) \times SU(2) \times U(1)_Y \times U(1)_R$ can be written as

$$W \sim \lambda^{n_{ij}^u} Q_i U_j^c H_2 + \lambda^{n_{ij}^d} Q_i D_j^c H_1 + \lambda^{n_{ij}^e} L_i E_j^c H_1 . \tag{3}$$

We follow the conventional normalization of R-charges: $R(W) = 2$. In eq. (3), the positive integers $n_{ij}^{u,d,e}$ are determined by

$$\begin{aligned}
q_i + u_i + h_2 - r_\chi n_{ij}^u &= 0 \\
q_i + d_i + h_1 - r_\chi n_{ij}^d &= 0 \\
l_i + e_i + h_1 - r_\chi n_{ij}^e &= 0 .
\end{aligned} \tag{4}$$

When the charge assingments for the quarks and leptons do not allow positive numbers $n_{ij}^{u,d,e}$, the corresponding Yukawa couplings are absent. This occurs whenever $(q_i + u_i + h_2)/r_\chi$ etc. are *negative* or *non-integers*. One can show that the determinants of the quark and lepton mass matrices fulfill

$$\begin{aligned}
\det M^u &\sim v_2^3 \lambda^{[\Sigma^u + 3h_2]/r_\chi} \\
\det M^d &\sim v_1^3 \lambda^{[\Sigma^d + 3h_1]/r_\chi} \\
\det M^e &\sim v_1^3 \lambda^{[\Sigma^e + 3h_1]/r_\chi}
\end{aligned} \tag{5}$$

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for *any values* of the R-charges [2]. Here $\Sigma^u \equiv \sum_i(q_i + u_i)$, $\Sigma^d \equiv \sum_i(q_i + d_i)$, $\Sigma^e \equiv \sum_i(l_i + e_i)$, and $v_{1,2} \equiv \langle H_{1,2} \rangle$. From eq. (5) we can draw two independent quantities which can be related to the anomalies of $U(1)_R$:

$$
\begin{aligned}
(\det M^u)(\det M^d) &\sim v_1^3 v_2^3 \lambda^{[\Sigma^u + \Sigma^d + 3(h_1 + h_2)]/r_\chi} \\
(\det M^e)/(\det M^d) &\sim \lambda^{[\Sigma^e - \Sigma^d]/r_\chi} .
\end{aligned}
\tag{6}
$$

The diagonalized fermion masses are known to satisfy the following ultraviolet relations [15]:

$$
\begin{aligned}
(m_u, m_c, m_t) &\sim v_2(\lambda^8, \lambda^4, 1) \\
(m_d, m_s, m_b) &\sim v_1 \lambda^x(\lambda^4, \lambda^2, 1) \\
(m_e, m_\mu, m_\tau) &\sim v_1 \lambda^x(\lambda^4, \lambda^2, 1) .
\end{aligned}
\tag{7}
$$

We have put the same factor λ^x for the $b - \tau$ unification: $m_b \simeq m_\tau$. Note that $\tan\beta \equiv v_2/v_1 = \lambda^x m_t/m_b$. Combining eqs. (6) and (7) two restrictions on the R-charges are obtained:

$$
\begin{aligned}
\Sigma^u + \Sigma^d &\simeq (3x + 18 + p)r_\chi - 3(h_1 + h_2) \\
\Sigma^e - \Sigma^d &\simeq qr_\chi .
\end{aligned}
\tag{8}
$$

Here two numbers p, q take into account the uncertainties in the ultraviolet mass relations (7): $|p|, |q| \leq 1$. The two quantities in eq. (8) can be expressed in terms of the anomalies $C_{3,2,1}$ of $U(1)_R$ with respect to $SU(3)$, $SU(2)$ and $U(1)_Y$. For the computation of the $U(1)_R$ anomalies we have to include the contribution from gauginos (with R-charge 1) as well as Hissinos, quarks and leptons. The anomalies are given by [16]:

$$
\begin{aligned}
C_3 &= \sum_i(2q_i + u_i + d_i) + 6 \\
C_2 &= \sum_i(3q_i + l_i) + (h_1 - 1) + (h_2 - 1) + 4 \\
C_1 &= \sum_i(\frac{1}{3}q_i + \frac{8}{3}u_i + \frac{2}{3}d_i + l_i + 2e_i) + (h_1 - 1) + (h_2 - 1) .
\end{aligned}
\tag{9}
$$

We do not consider the other anomalies like $U(1)_Y$–$U(1)_Y$–$U(1)_R$ or $U(1)_R^3$ etc.. The former one entails specific R-charge assignments for quarks and leptons and the latter requires full spectrum including hidden supersymmetry breaking sector [16] which we do not address in

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[2]In ref. [8] it was noted that this relation holds for integer or non-integer values of the R-charges.

this paper. From eq. (9) one gets two independent combinations

$$C_3 = \Sigma^u + \Sigma^d + 6$$
$$C_1 + C_2 - \frac{8}{3}C_3 = 2\Sigma^e - 2\Sigma^d + 2(h_1 + h_2) - 16 \ . \tag{10}$$

Comparing eqs. (8) and (10), one finds

$$C_3 = (3x + 18 + p)r_\chi - 3(h_1 + h_2) + 6$$
$$C_1 + C_2 - \frac{8}{3}C_3 = 2qr_\chi + 2(h_1 + h_2) - 16 \ , \tag{11}$$

from which the main results can be drawn.

In the case of $h_1 + h_2 = 0$ corresponding to $\mu \sim m_{3/2}$ (1) two anomalies in eq. (11) cannot vanish simultaneously within the allowed uncertainties in p and q. Therefore one has to rely on the Green-Schwarz mechanism of anomaly cancellation [6]. The Green-Schwarz mechanism implies $\alpha_3 C_3 = \alpha_2 C_2 = \alpha_1 C_1$ for the ultraviolet gauge coupling constants $\alpha_i = g_i^2/4\pi$. Thus the canonical normalization of the gauge couplings $\alpha_3 = \alpha_2 = \frac{5}{3}\alpha_1$ requires the second quantity in eq. (11) to vanish: $C_1 + C_2 - \frac{8}{3}C_3 = 0$ [7]. For this, one needs $q \neq 0$ and $r_\chi = 8/q$.

The μ-term can result also from the following non-renormalizable terms in the Kähler potential:

$$K \sim \frac{\chi^*}{M_P}H_1 H_2 \quad \text{or} \quad \frac{\chi}{M_P}H_1 H_2 \ . \tag{12}$$

The corresponding μ-parameter is then given by $\mu \sim \lambda m_{3/2}$. We will not consider the case with further suppressed values of μ since it becomes too small. Two terms in eq. (12) are allowed when $h_1 + h_2 = kr_\chi$ with $k = \pm 1$, respectively. Once again one finds that the anomaly cancellation can be achieved only by the Green-Schwarz mechanism. The canonical normalization requires the R-charge $r_\chi = 8/(k + q)$. Similar conclusions can be drawn also when $\lambda^2 \sim \langle \chi \rangle/M_P$ is taken as the expansion parameter.

In conclusion, we have shown that a gauged horizontal $U(1)$ R-symmetry requires the Green-Schwarz mechanism for anomaly cancellation in order to explain both the observed fermion mass hierarchies and the appearance of the μ-parameter. The μ-parameter comparable to the gravitino mass $m_{3/2}$ is acceptable only if the ultraviolet mass relation $m_e m_\mu m_\tau/m_d m_s m_b \simeq \lambda^q$ ($q = \pm 1$) is assumed. On the contrary, a suppressed value of $\mu \sim \lambda m_{3/2}$ is consistent with

any value of q. Our conclusion is based on the minimal model with only one expansion parameter generated by the vacuum expectation value of an extra singlet and on the assumption of the canonical normalization of the gauge coupling unification $g_3^2 = g_2^2 = \frac{5}{3}g_1^2$.

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